## Computer Number Systems

All computers are electronic devices and can ultimately do one thing: detect whether an electrical signal is "on" or "off". Therefore, the earliest computer scientists realized that the binary number system which is base 2 and has only the digits 0 and 1 was better to use than the decimal number system which is base 10 and has digits $0,1,2$, ..., 9 .

They called each binary digit a "bit" and used them to represent ON or TRUE with a 1 and OFF or FALSE with a 0 .

## Octal and Hexadecimal Numbers

Because binary numbers could get very long even for small decimal numbers, they grouped 3 binary digits together to form one base 8 digit and called that octal and grouped 4 binary digits together to form one base 16 digit and called that hexadecimal. The following table shows equivalent values in these bases:

Three digits ( 0 for left-most) $0000=0$
$0001=1$
$0010=2$
$0011=3$
$0100=4$
$0101=5$
$0110=6$
$0111=7$

Four digits (needed for base 16)
$1000=8$
$1001=9$
$1010=\mathrm{A}(10)$
$1011=B(11)$
$1100=C(12)$
$1101=\mathrm{D}(13)$
$1110=E(14)$
$1111=F(15)$

As you can see, the largest value that can be represented with 3 binary digits (when the first digit is a 0 ) is 7 and the largest value with 4 bits is 15 . However, because all digits must be single characters, the letters $A-F$ are used to represent the values 10 to 15 . Therefore, in base 8 , the digits must be 0 to 7 and in base 16 the digits must be $0-9, A-F$.

## Grouping Binary Digits for Octal and Hexadecimal Numbers

An easy way to convert between bases 2,8 , and 16 is by grouping bits together and using the table above.
$1001010110_{2}=001001010110_{2}=1126_{8}$
and
$1001010110_{2}=001001010110_{2}=256_{16}$
Notice that grouping is always done starting with the last digit.

Here are two other examples:
A) Write $1010111011110101_{2}$ in octal:

Divide the binary number into groups of 3 from right to left.
Convert each grouping to its octal value. $0010101110111101012=127365_{8}$
B) Write $1010111011110101_{2}$ in hexadecimal:

Divide the binary number into groups of 4 from right to left.
Convert each grouping to its hexadecimal value.

$$
1010111011110101_{2}=\mathrm{AEF} 5{ }_{16}
$$

## Using Expanded Notation to Find Base 10 Values

In addition, expanded notation in the decimal number system can represent different numbers by using powers of 10 :

4,381

$$
\begin{aligned}
4,381 & =4 \times 1000+3 \times 100+8 \times 10+1 \\
& =4 \times 10^{3}+3 \times 10^{2}+8 \times 10^{1}+1 \times 10^{0} \\
50,070 & =5 \times 10^{4}+0 \times 10^{3}+0 \times 10^{2}+7 \times 10^{1}+0 \times 10^{1} \\
& =50000+70
\end{aligned}
$$

Remember that any non-zero number raised to the 0 power equals 1 and 0 multiplied by anything is 0 .

All other bases work the same way as follows:

$$
\begin{array}{rlllll}
1101_{2} & =1 \times 2^{3} & +1 \times 2^{2} & +0 \times 2^{1} & +1 \times 2^{0} & \\
& =8 & +4 & +0 & +1 & =13_{10}
\end{array}
$$

(Compare this value with the above table.)

$$
\left.\begin{array}{rlrl}
175_{8}=1 \times 8^{2}+7 \times 8^{1} & +5 \times 8^{0} & \\
& =1 \times 64+7 \times 8 \quad+5 \times 1 & \\
& =64 & +56 & +5
\end{array}\right)
$$

## Using Hexadecimal Numbers to Represent Colors

Computers use hexadecimal numbers to represent various colors in computer graphics because all computer screens use combinations of red, green, and blue light or RGB to represent thousands of different colors.

Two digits are used for each so the hexadecimal number "\#FF0000" represents the color red, "\#00FF00" represents green, and "\#0000FF" represents blue. The color black is " $\# 000000$ " and white is "\#FFFFFF".

The hash tag or number sign is used to denote a hexadecimal number. $\mathrm{FF}_{16}=\mathrm{F}$ (15) x $16+F(15) \times 1=240+15=255_{10}$ so there are 0 to 255 or 256 different shades of each color or $256^{3}=16,777,216$ different colors.

The following web site has nearly every color name, along with its hex code and decimal values:

## https://www.rapidtables.com/web/color/RGB Color.html

For example "salmon" is "\#FA8072" which represents the decimal numbers 250 (hex FA), 128 (hex 80), and 114 (hex 72).

## Adding and Subtracting in Other Bases

When adding numbers in base 10, you have learned to carry a 1 to the next place when the answer is 10 or more and keep the value that is more than 10 . For example, $15+48=63$ because $5+8=13$ so you keep the 3 and carry the 1 . When subtracting numbers, you have learned to borrow a 1 from the next place and add 10 to the current digit. For example, 75-48

The same rules can be used in any base by using the number of the base instead of 10.

Some simple examples are:

$$
\begin{array}{lll}
0_{2}+0_{2}=0_{2} & 0_{2}+1_{2}=1_{2} & 1_{2}+1_{2}=10_{2} \\
7_{8}+1_{8}=10_{8} & F_{16}+1_{16}=10_{16} & F_{16}+1_{16}=100_{16}
\end{array}
$$

Other examples in base 8 and 16 are:
A) 6458
B) A 6 B 516
C) 6458
D) $\mathrm{A} 6 \mathrm{~B} 5_{16}$
$+372_{8}$
$1237_{8}$

| +4 C1F $_{16}$ | $-372_{8}$ |
| :---: | :---: |
| -------------- |  |
| F2D4 |  |

-4 C1F $_{16}$
$--------19{ }_{16}$

Remember that every letter in base 16 must be changed to its equivalent value first.

In problem $B, 5+F(15)=20$ so keep $20-16=4$ and carry a 1 so $A(10)+4+1=15$ (F). In problem $D$, borrow 16 from the $B$ and make it an $A(10)$. Then, $16+5=21-F$ $(15)=6$. The same concept is used in octal by carrying or borrowing 8 .

## References

http://csunplugged.org/binary-numbers/ http://www.mathmaniacs.org/lessons/01-binary/ https://educators.brainpop.com/bp-topic/binary/ https://en.wikipedia.org/wiki/RGB color model

## Sample Problems

| What is the value of $11101001011_{2}$ in base 16 ? | By grouping digits, the binary number $11101001011_{2}=01110100{1011_{2}=74 B_{16} .}^{2}$ |
| :---: | :---: |
| What is the value of $1 A C E_{16}+456_{16}$ in hexadecimal? | Using the method shown above, E (14) + 6 $=20$ so carry the 1 and keep the 4 . $C(12)+$ $5+1=18$ so carry the 1 and keep the 2. A $(10)+4+1=15$ which is $F$ so there is nothing to carry. The answer is 1F24 ${ }_{16}$. |
| What is the value of $135_{8}$ in base 10 ? | $\begin{aligned} 1358 & =1 \times 8^{2}+3 \times 8^{1}+5 \times 8^{0} \\ & =64+24+5=93_{10} . \end{aligned}$ |
| What is the value of $1101011_{2}$ in base 10? | By using expanded notation starting on the right, the value is $1 \times 2^{0}+1 \times 2^{1}+0 \times 2^{2}+1$ $\times 2^{3}+0 \times 2^{4}+1 \times 2^{5}+1 \times 2^{6}=1+2+8+$ $32+64=107_{10}$. |
| How many 1 s are in the binary representation of 43278? | $4327_{8}=100011010111_{2}$ so there are 7 bits that are 1 s and 5 bits that are 0 s . |


| On the RGB color table, the color "sky blue" is the hexadecimal number '\#38B0DE'. What is the decimal value for the blue component? | The RED component is ' 38 ', the GREEN component is ' $\mathrm{B0}$ ', and the BLUE component is ' DE '. <br> Therefore, $D E_{16}=13 \times 16+14 \times 1=208+$ $14=222_{10}$. |
| :---: | :---: |
| What is the value of 1CE2 ${ }_{16}-9 F 6_{16}$ in hexadecimal? | For the last two digits, you can't subtract 2 from 6 so borrow and make E a D. You can't subtract $F$ from $D$ so borrow and make $C$ a B. Borrow 16 in each case. Then $2+16-6$ $=12(C) . D(13)+16-F(15)=14(E) . B(11)-$ $9=2$. The answer is $12 E C_{16}$. |
| Which of the following is the largest number? <br> a) $\mathrm{AB}_{16}$ <br> b) $10101100_{2}$ <br> c) 2518 | $\begin{aligned} & A B_{16}=10101011_{2}=10^{*} 16+11=171_{10} \\ & 10101100_{2}=4+8+32+128=172_{10} \\ & 251_{8}=10101101_{2}=2^{*} 64+5^{*} 8+1=169_{10} \end{aligned}$ <br> Either way, the largest number is (b). |
| What is the average of the following three numbers in base 10 ? $10011_{2}, 21_{8}, 1 \mathrm{E}_{16}$ | $\begin{aligned} & 10011_{2}=1 * 1+1 * 2+0 * 4+0 * 8+1 * 16=19 \\ & 21_{8}=2 * 8+1=17 \\ & 1 E_{16}=1 * 16+14=30 \\ & (19+17+30) / 3=66 / 3=22_{10} \end{aligned}$ |
| Which of the following numbers has the least number of 1 's in their binary representation? <br> A) $B A D_{16}$ <br> B) $\mathrm{FED}_{16}$ <br> C) $\mathrm{CAB}_{16}$ <br> D) $A C E E_{16}$ | $B A D_{16}=101110101101_{2}$ which is 8 bits. $F_{E D}{ }_{16}=111111101101_{2}$ which is 10 bits. $C A B_{16}=110010101011_{2}$ which is 7 bits. $A D E_{16}=101011011110_{2}$ which is 8 bits. <br> Therefore, the answer is letter (C). |

